

With the rectangles stored in the computer as files, the machine is in the position to manipulate them. It can make them into nets and search for groups of them which meet certain prerequisites. John C. Wilson (JCW) who was under the supervision of WTT would earn the degree of Doctor of Philosophy for the discovery that simple perfect squares having the structure of that shown in Figure 36 (with a characteristic single stair step line running from border to border) could, through the manipulation of their nets, be dissected into two perfect rectangles. Simple perfect squares with such a feature may said to be of **Wilsonian structure**. Another way of expressing the concept is to say that such squares are composed of two **perfect squared hexagons** (any roughly L-shaped area [with 6 sides thus the term hexagon] that is completely subdivided into squares, none of which is the same size).

A general overview of the methods of JCW are as follows:

- 1.) It is possible to proceed from a simple perfect square of Wilsonian structure to the generative rectangles or equally possible to proceed from the generative rectangles (the identification of which requires a computer search) to a simple perfect square of Wilsonian structure.
- 2.) Since the simple perfect rectangles through order 19 are already known (thanks to CJB) and the simple perfect squares are not, the order of attack is clear. One proceeds from the generative rectangles to the squares. The point is that this can be a two-way street in that once the square is known, it is always possible to recapture the generative rectangles. The procedure uses only two generative rectangles.
3. To proceed from the rectangles to the Wilsonian square, a computer search is initiated to identify a suitable pair of perfect rectangles from the existing data base on file in the machine. The reader interested in the particulars of just how to program a machine to make such a search is referred to Wilson's dissertation for the technical details. It is cited in the bibliography.
- 4.) These rectangles are chosen so when placed side by side (something done as an aid to human visualization of the problem) that if L_1 and S_1 respectively are taken to be the long and short sides of the first rectangle and L_2 and S_2 the corresponding sides of the second rectangle that $(S_2 \times L_1) + (S_1 \times L_2) = N^2$, where N^2 is the area of the future Wilsonian square. Thus N is the side of the square to be constructed.
- 5.) At the same time, an analysis of the nets is conducted to determine if the choice of the pair meets the prerequisites of the technique. That is to say, poles are identified such that partial side L_1 , pL_1 , and partial side L_2 , pL_2 , fulfill the following equality: $(pL_1 \times L_2) - (pL_2 \times L_1) = (C_L \times N)$ where C_L is the level of change in the hexagons, that is the height of the rise in the stairstep.

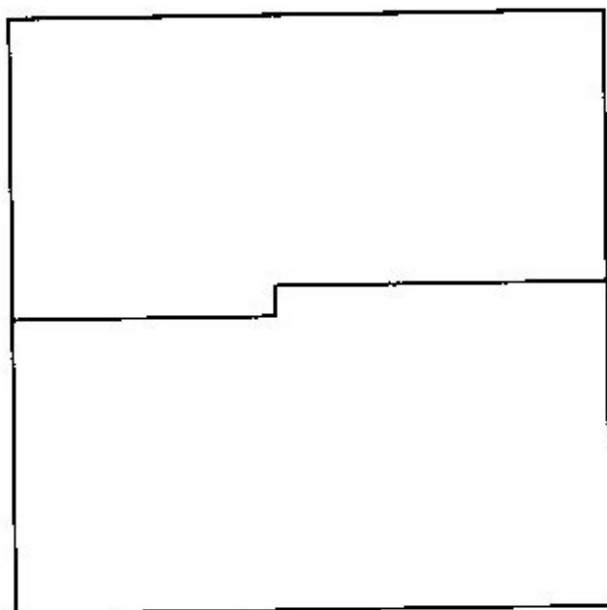


Figure 36. Stair step structure of
J. C. Wilson's squares.



6.) If conditions 4 and 5 are met then a suitable candidate has been identified and one can proceed. Failing this, the pair is rejected as unsuitable and the search is continued.

7.) Assuming a suitable pair has been identified, it is possible to specify the poles within the net of each rectangle so as to make it a hexagon. The true value of nets is apparent here, for what may be specified as poles may surprise many first time readers. In doing so, it will be recalled that the Kirchhoff laws require that the values on one side of a line equate to those on the other sides except at the edge. In this case, the critical pole will not be the edge, rather it will be an internal line perpendicular to the edge which is coalesced. This procedure is applied to both rectangles and critical poles **a** and **b** are identified.

8.) Hex-nets are now drawn and placed side by side. Hex-nets are triangular in shape so the appropriate bases are parallel. The nets are thus arranged so that by fusing the positive and negative poles into a single terminal each (from two such in each triangular net) that a single polar net is now formed for the Wilsonian square.

9.) This p-net is diamond or rhombic in shape which is typical for nets of squares and rectangles. From this it is possible to, as we did for 9: 32 X 33 (ZM), solve a system of simultaneous equations to get the values for the currents in the system and from this to draw up the square using as elemental sides the currents of the wires of the p-net of the Wilsonian square as the elements.

All this, while just "ho hum" to the professional reader, may be a bit confusing or too much to take in for first time readers. Therefore let's illustrate it by "cook booking" an example. From this, first time readers may be surprised to discover the skills to grasp the subject have already been developed *en route*. From the illustrated example, one can then generalize to other cases. For the sake of argument, it is given that the computer has identified a suitable pair of rectangles for us. We have in hand, two simple perfect rectangles identified by a computer search, 14: 705 X 832 and 11: 127 X 209. Right away, we know that the outcome will be a 25th order simple (hopefully) perfect square of Wilsonian structure, because no element is lost nor gained by the process and $14 + 11 = 25$. Recall $(S_2 \times L_1) + (S_1 \times L_2) = N^2$. So, $(127 \times 832) + (705 \times 209) = 503^2$. Also recall $(pL_1 \times L_2) - (pL_2 \times L_1) = (C_L \times N)$. Thus, $(424 \times 209) - (92 \times 832) = (24 \times 503)$. This confirms that the machine's choice is suitable as to the prerequisites to success with this method. In this case, pL_1 is 424 and pL_2 is 92 as will be seen from figure 37 (purposefully depicted as a crude sketch of the two suitable rectangles which one might hastily make in practice). Thus, pL_1 is a consequence of identifying critical pole **a** and pL_2 is a consequence of identifying critical pole **b**. Recall that in practice the machine would do the work for us, but we will do the work from here to ensure that the necessary skills to use the technique are clearly understood. It is important to know what the

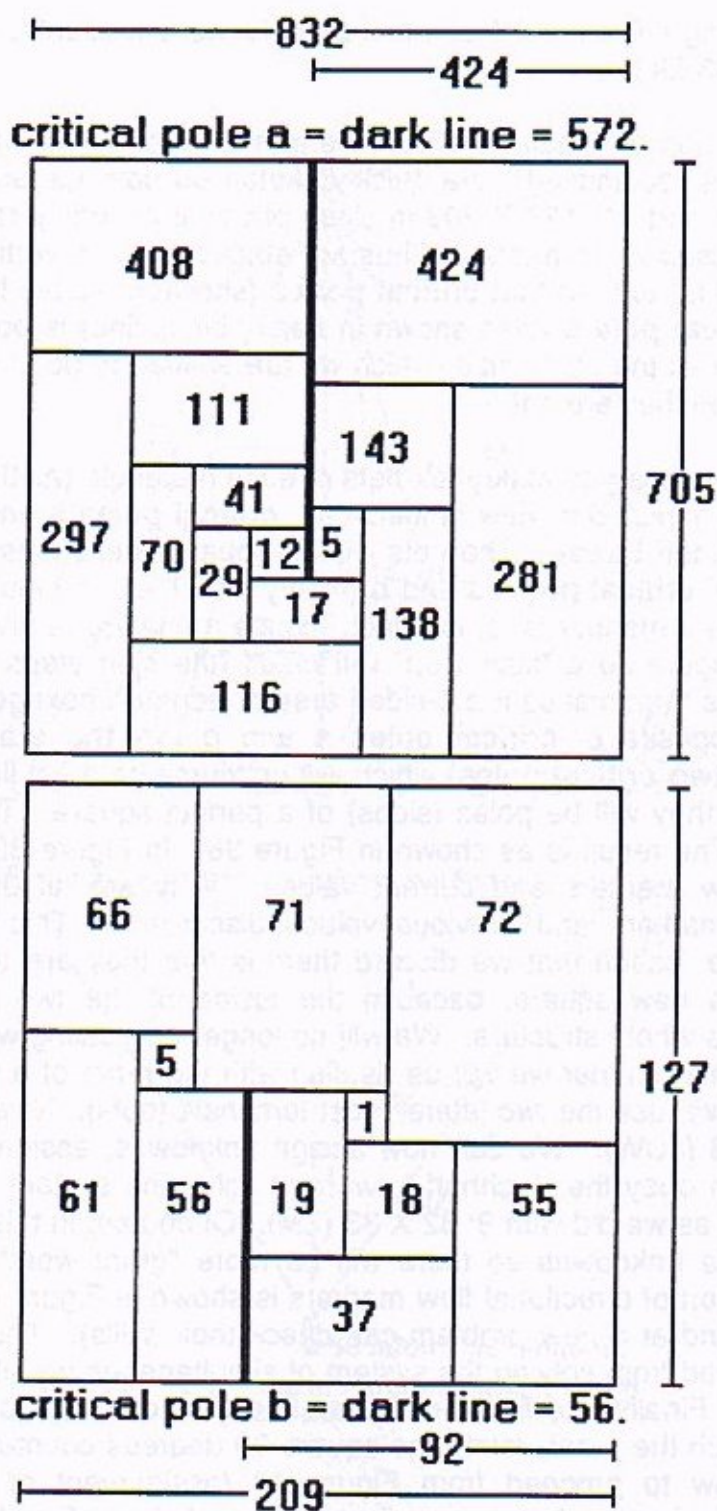


Figure 37. Using Wilson's method.

machine is doing and why. What it has done for us is to identify what will be the poles of 25: 503 (JCW).

The first step we shall take from here is to refer to Figure 37 (the crude sketches of the rectangles). We quickly sketch up both generative rectangles 14: 705 X 832 and 11: 127 X 209 in close physical proximity (as an aid to our thought processes in the matter). Thus, we stack them in a vertical manner with 14: 705 X 832 on top, so that **critical pole a** (shown in heavy black line) is top most and **critical pole b** (also shown in heavy black line) is bottom most. We will show them as the same size (which we are entitled to do in crude sketches) but bear in mind they are not.

Now it is necessary to make hex-nets of each rectangle (as they must appear to be properly depicted in view of identified **critical poles a** and **b** [so they will obey the Kirchhoff Laws]). The nets will be squared hexagons because of the identification of **critical poles a** and **b** (heavy dark lines in Figure 37). To form the net in such a manner is to, in effect, cause a change of level in the side of the resulting figure so a "stair step" will result (the stair steps will fit precisely together). This then makes it a 6-sided area or squared hexagon, with the stair step being opposite of **critical poles a** and **b** (so the stair step occurs **between the two critical poles**) which will of course be a flat line or side in the final result as they will be poles (sides) of a perfect square. The hex-nets are now drawn. The result is as shown in Figure 38. In Figure 39, we discard all directional flow markers and current values. Why are all of the directional current flow markers and previous values discarded? This is an insightful question. The reason that we discard them is that they are almost certain to change in the new square, because the fusion of the two nets will cause changes to the whole structure. We will no longer be dealing with two hex-nets in close proximity, rather we will be dealing with the p-net of a squared square. In Figure 40, we fuse the two lateral most terminals (dots). Now we have the p-net of 25: 503 (JCW). We can now assign unknowns, assign directional flow markers which obey the Kirchhoff Laws and solve the system of simultaneous equations just as we did with 9: 32 X 33 (ZM). Of course, in this case, there will be many more unknowns so there will be more "grunt work" involved. The correct depiction of directional flow markers is shown in Figure 41 (so that those trying their hand at a new problem can check their skills). The correct current values (obtained from solving the system of simultaneous equations) are shown in Figure 42. Finally, the finished square is depicted in Figure 43 in canonical form. To match the p-net, rotate the square 90 degrees counter-clockwise. If it is unclear how to proceed from Figure 40 (assignment of directional flow markers, assignment of unknowns and the solution of a large system of simultaneous equations) then a review of the Kirchhoff Laws and basic algebraic procedures involved is both indicated and strongly recommended [as it is beyond the scope of this book]).

The Wilson method was a major theoretical contribution to the field, but not the only one to occur in this period. We pick up the tale in the 1960's. Another

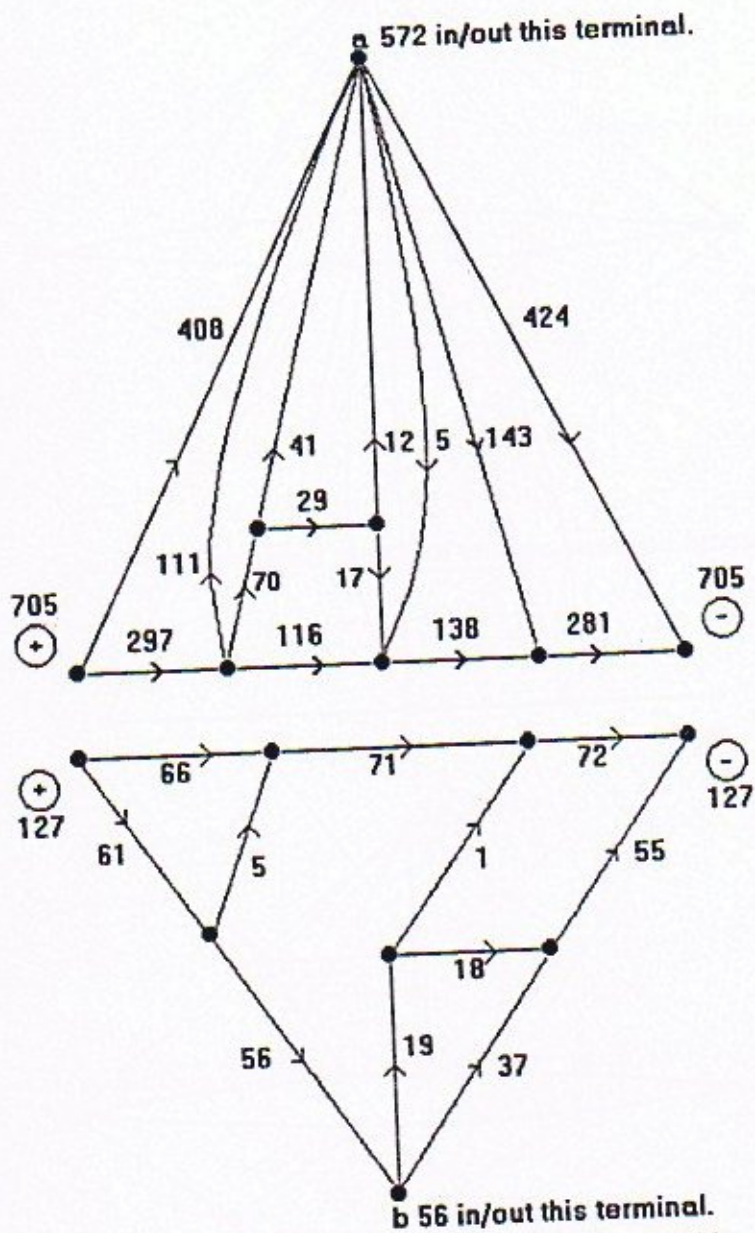


Figure 38. Polar nets of Figure 37 with a and b as critical poles.

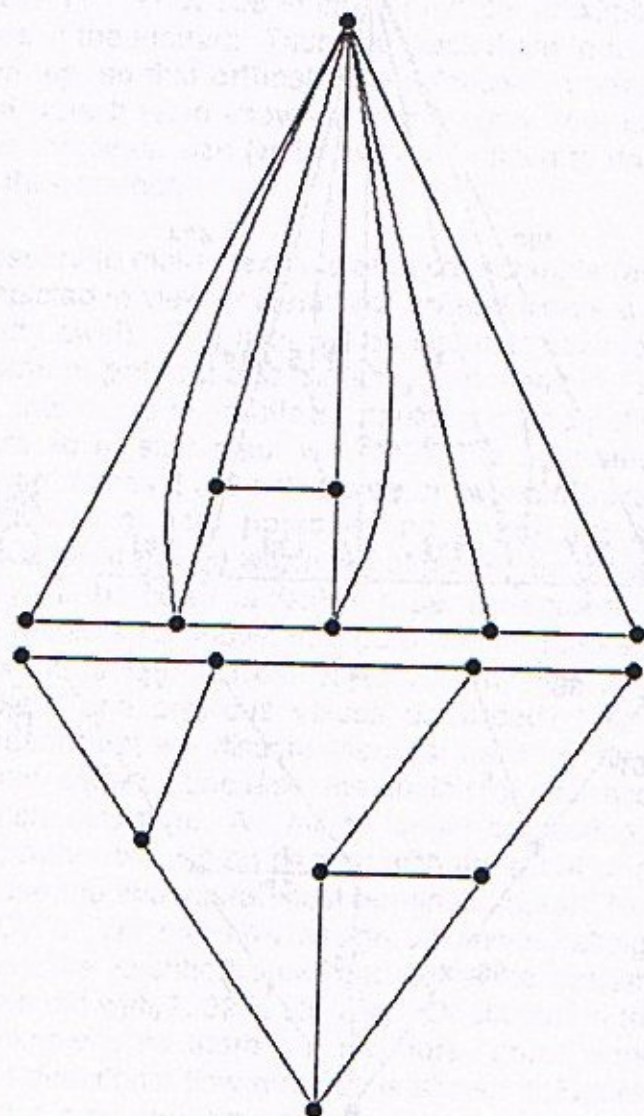


Figure 39. The two nets prior to fusion of poles.

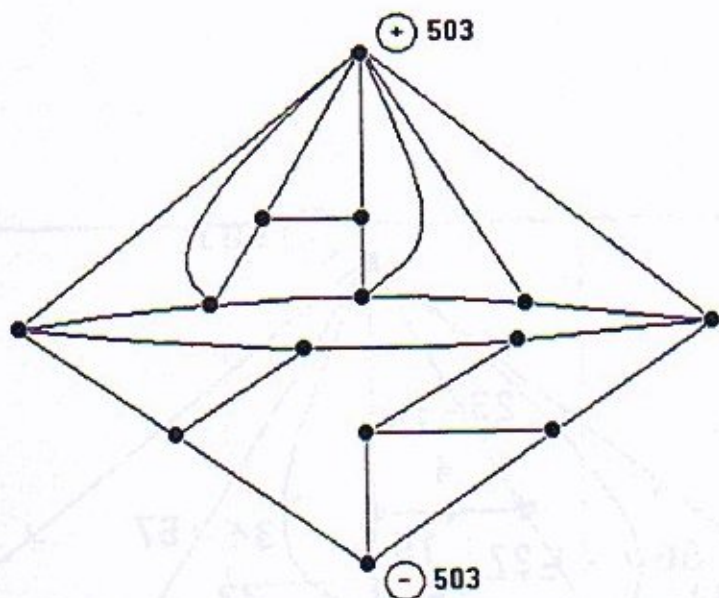


Figure 40. The fused p-nets of the two squared hexagons.

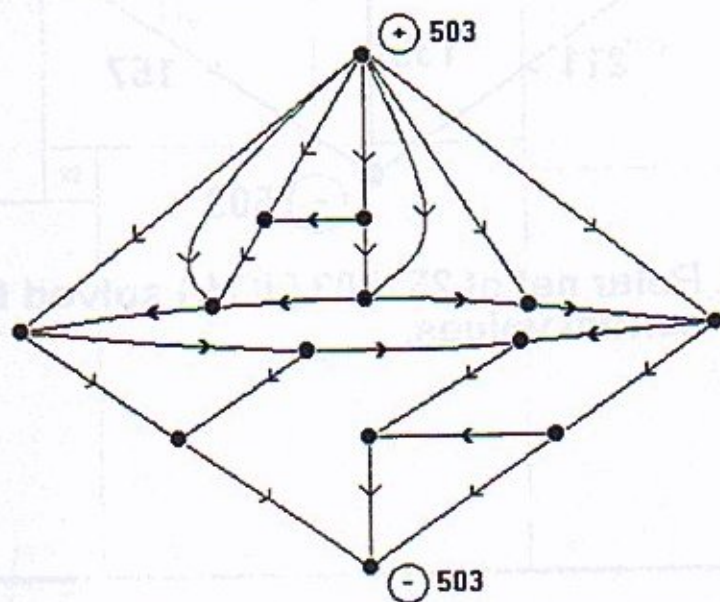


Figure 41. Polar net of 25: 503 (JCW) with directional flow of current indicated.

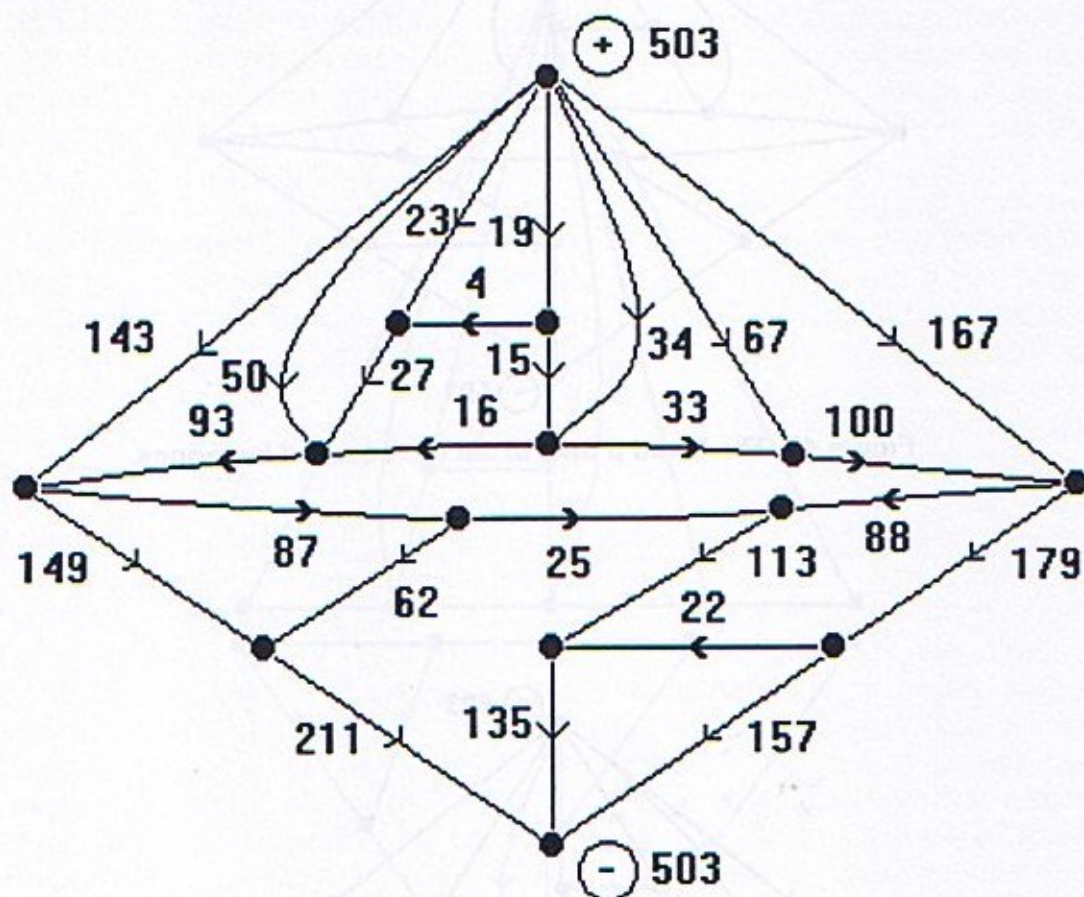
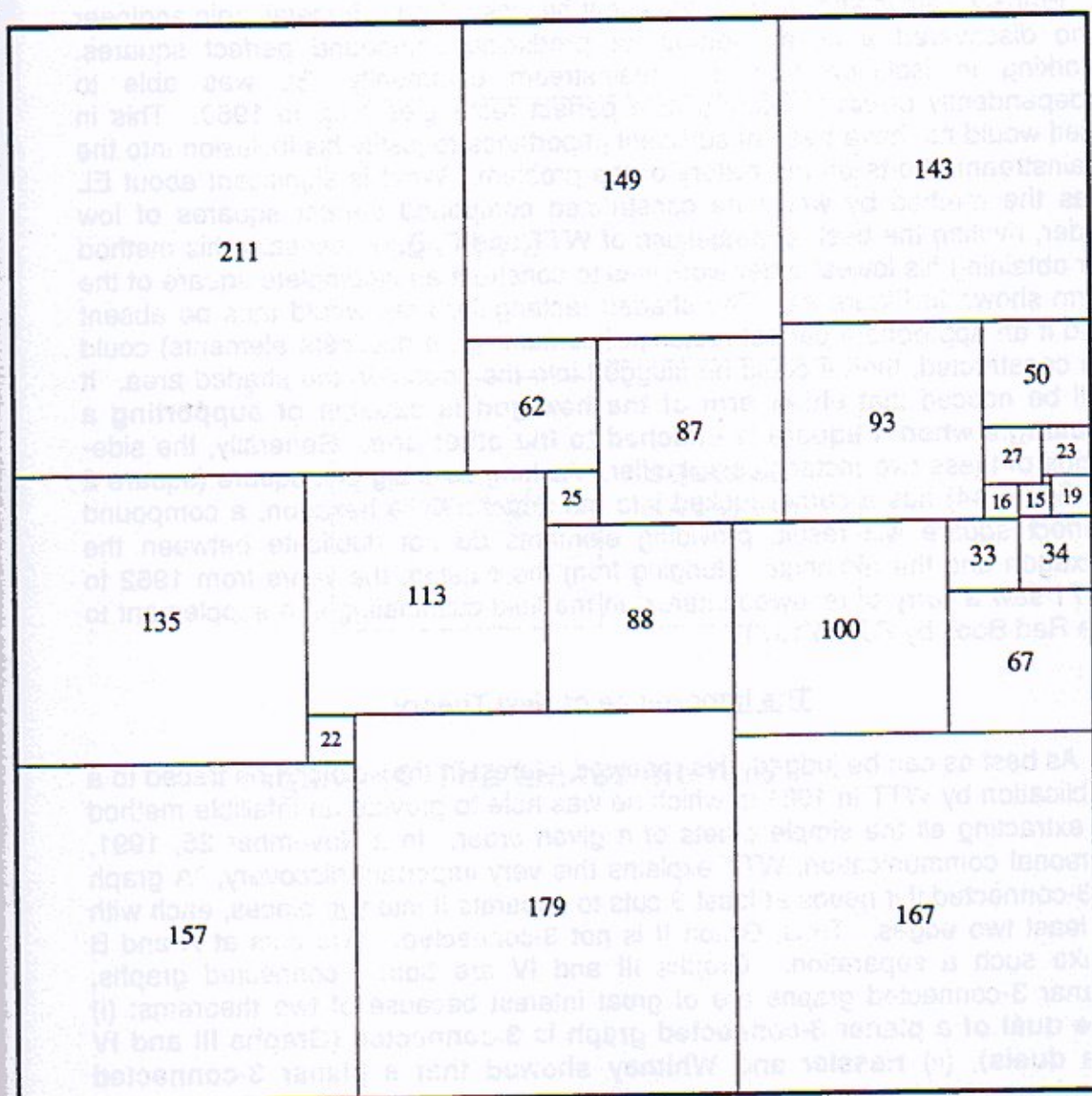


Figure 42. Polar net of 25: 503 (JCW) solved for unknown current values.

503



503

Figure 43. The final solution - 25: 503 (JCW), a simple perfect square of Wilsonian architecture as found by a computer search.